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Exothermic Transition to a Future Susy Universe

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Abstract The Higgs sector of the MSSM may be extended to solve the μ problem by the addition of a gauge singlet scalar field. We consider an extended Higgs model. For simplicity we consider the case where all the fields in the scalar sector are real.

We analyze the vacuum structure of the model. We address the question of an exothermic phase transition from a broken susy phase with electroweak symmetry breaking (our current universe) to an exact susy phase with electroweak symmetry breaking (future susy universe).

Keywords Supersymmetry · Singlet extended SUSY Higgs Models · Phase transition

1 Introduction

Supersymmetric extensions of the Standard Model (SM) are apparently the best candidate for new physics at low energy. They provide an elegant solution to hierarchy problem of the SM. They have neutralino as cold dark matter candidate. Accurate gauge coupling unification is achieved in the minimal supersymmetric extension of the Standard Model (MSSM) [1–3] as well.

But the MSSM suffers from the μ-problem [4, 5]. The bilinear supersymmetric Higgs mass term $\mu H_d H_u$ in the superpotential, where H_d and H_u are one pair of Higgs doublets, does not violate supersymmetry and gauge symmetry. Then the natural scale for μ is about Planck scale. However in order to get the weak scale correctly with unnatural cancellation we need μ to be about TeV scale.

One solution for this so-called μ-problem is to substitute a VEV of an extra gauge singlet field for the parameter μ. In the past thirty years various singlet extensions of the MSSM has been considered [6–9].

A singlet extension of MSSM with mirror symmetry is [10] defined by the superpotential

$$W = \lambda(S(H_u.H_d - v^2) + \tilde{S}(\tilde{H}_u.\tilde{H}_d - v^2) + \mu_0 S\tilde{S}). \tag{1}$$

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In this model Higgs singlet field is coupled to a mirror world (hidden sector) indicated by tildes. And the dot product of the two Higgs doublets is defined by

$$H_u.H_d = H_u^0 H_d^0 - H_u^+ H_d^-. \tag{2}$$

As we do not consider the possibility that the charged Higgs fields could acquire vacuum expectation values we suppress occurrence of charged Higgs fields. So the product in (2) is taken to be equivalent to the product of the neutral fields.

In a recent paper [11] we showed that a true symmetry breaking minimum does not exist. And the model has two critical points, where at these points all first derivatives of the scalar potential with respect to the fields vanish:

Solution 1: Exact susy with Electroweak Symmetry Breaking (EWSB)
Solution 2: Exact susy with no EWSB.

In our analysis we neglected the phases in the Higgs sector. We also did not include the soft susy breaking terms.

In this work we assume that the fields and the parameters in the Lagrangian of the Higgs sector are real. We attempt a phenomenological treatment of extended Higgs model and we give a partial analysis of soft susy breaking terms [12].

The motivation for the present work is as follows:

(i) To show that a true susy breaking minima is attainable by addition of soft Higgs masses.
(ii) And to study the case of an exothermic phase transition to a future susy universe [13–15].

In solution (1) the Higgs vacuum expectation values in the broken susy phase is ν. In this work we seek solution in the broken susy phase with Higgs vacuum expectation value ν_0. We consider the case of $\nu_0 > \nu$. So an exothermic transition to exact susy could lead in a simple way to a world supporting atoms and molecules [10]. One also must assume that when the transition to exact susy occurs these soft terms will vanish.

In section two we describe the model. We obtain the critical point condition on the parameters and vevs and we obtain a symmetric solutions where the vevs of the scalar fields in the real world is equal to that of their associated fields in the hidden sector. In section three we obtain the Higgs mass squared matrix of our solution. In section four we discuss the case of exothermic transition to a future susy universe and we show that copious solutions exist. And finally in section five we present our conclusions.

2 A Model with Mirror Symmetry

The F term in the scalar potential in any supersymmetric model is derivable from the superpotential by

$$V_F = \sum_\phi \left| \frac{\partial W}{\partial \phi} \right|^2. \tag{3}$$

So for the neutral fields the F term of the scalar potential from the superpotential (1) is

$$\begin{aligned} V_F = \lambda^2[&|H_u H_d - \nu^2 + \mu_0 \tilde{S}|^2 + |S|^2(|H_u|^2 + |H_d|^2) + |\tilde{H}_u \tilde{H}_d - \nu^2 + \mu_0 S|^2 \\ &+ |\tilde{S}|^2(|\tilde{H}_u|^2 + |\tilde{H}_d|^2)] \end{aligned} \tag{4}$$

The D term in the potential is

$$V_D = \frac{g_1^2+g_2^2}{8}[|H_d|^2-|H_u|^2]^2 + \frac{g_2^2}{2}[|H_d|^2|H_u|^2-|H_uH_d|^2] + \frac{g_1^2+g_2^2}{8}[|\tilde{H}_d|^2-|\tilde{H}_u|^2]^2 + \frac{g_2^2}{2}[|\tilde{H}_d|^2|\tilde{H}_u|^2-|\tilde{H}_u\tilde{H}_d|^2], \tag{5}$$

where g_1 and g_2 are the $U(1)$ and $SU(2)$ gauge couplings.

The general structure of soft susy breaking term has a complicated form [13]. Here we only consider the soft mass squared Higgs term, therefore

$$V_s = \lambda^2[m_S^2|S|^2 + m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_{\tilde{S}}^2|\tilde{S}|^2 + m_{\tilde{H}_u}^2|\tilde{H}_u|^2 + m_{\tilde{H}_d}^2|\tilde{H}_d|^2]. \tag{6}$$

Hence, our simplified model is

$$V = V_F + V_D + V_s. \tag{7}$$

For simplicity we will ignore phases in the Higgs sector as well. The Vacuum expectation values of the Higgs are given by

$$\langle H_u\rangle = v_1, \qquad \langle H_d\rangle = v_2. \tag{8}$$

Similarly for the Higgs in the mirror world we have

$$\langle \tilde{H}_u\rangle = \tilde{v}_1, \qquad \langle \tilde{H}_d\rangle = \tilde{v}_2. \tag{9}$$

$$\langle S\rangle = S_0 \quad \text{and} \quad \langle \tilde{S}\rangle = \tilde{S}_0. \tag{10}$$

By minimizing the scalar potential we obtain

$$S_0(v_1^2+v_2^2) + \mu_0(\tilde{v}_1\tilde{v}_2 - \nu^2 + \mu_0 S_0) + m_S^2 S_0 = 0, \tag{11}$$

$$\tilde{S}_0(\tilde{v}_1^2+\tilde{v}_2^2) + \mu_0(v_1v_2 - \nu^2 + \mu_0 \tilde{S}_0) + m_{\tilde{S}}^2 \tilde{S}_0 = 0, \tag{12}$$

$$v_2(v_1v_2 - \nu^2 + \mu_0\tilde{S}_0) + v_1 S_0^2 + \frac{v_1}{4\lambda^2}(g_1^2+g_2^2)(v_1^2-v_2^2) + m_{H_u}^2 v_1 = 0, \tag{13}$$

$$v_1(v_1v_2 - \nu^2 + \mu_0\tilde{S}_0) + v_2 S_0^2 + \frac{v_2}{4\lambda^2}(g_1^2+g_2^2)(v_2^2-v_1^2) + m_{H_d}^2 v_2 = 0, \tag{14}$$

$$\tilde{v}_2(\tilde{v}_1\tilde{v}_2 - \nu^2 + \mu_0 S_0) + \tilde{v}_1 \tilde{S}_0^2 + \frac{\tilde{v}_1}{4\lambda^2}(g_1^2+g_2^2)(\tilde{v}_1^2-\tilde{v}_2^2) + m_{\tilde{H}_u}^2 \tilde{v}_1 = 0, \tag{15}$$

$$\tilde{v}_1(\tilde{v}_1\tilde{v}_2 - \nu^2 + \mu_0 S_0) + \tilde{v}_2 \tilde{S}_0^2 + \frac{\tilde{v}_2}{4\lambda^2}(g_1^2+g_2^2)(\tilde{v}_2^2-\tilde{v}_1^2) + m_{\tilde{H}_d}^2 \tilde{v}_2 = 0. \tag{16}$$

A useful parameter in the Higgs sector is $\tan\beta = \frac{v_1}{v_2}$. In this work we seek solutions with $\tan\beta \neq 1$.

Solution 3:

$$v_1 = \tilde{v}_1, \qquad v_2 = \tilde{v}_2, \quad \text{and} \quad S_0 = \tilde{S}_0. \tag{17}$$

From (11)–(16) we find that for this solution

$$m_{H_u}^2 = m_{\tilde{H}_u}^2, \qquad m_{H_d}^2 = m_{\tilde{H}_d}^2, \quad \text{and} \quad m_S^2 = m_{\tilde{S}}^2. \tag{18}$$

This will then corresponds to a broken susy phase with EWSB.

3 Higgs Mass Squared Matrix

In this section we compute the Higgs mass matrix for the solution. In the space of $H_u, H_d, S, \tilde{H}_u, \tilde{H}_d, and \tilde{S}$ this mass squared matrix is obtained from the second derivative of the scalar potential. And for this case is

$$M^2 = \begin{pmatrix} a_1 & b_1 & c_1 & 0 & 0 & d_1 \\ b_1 & a_2 & c_2 & 0 & 0 & d_2 \\ c_1 & c_2 & e_1 & d_1 & d_2 & 0 \\ 0 & 0 & d_1 & a_1 & b_1 & c_1 \\ 0 & 0 & d_2 & b_1 & a_2 & c_2 \\ d_1 & d_2 & 0 & c_1 & c_2 & e_1 \end{pmatrix} \tag{19}$$

The elements of this matrix are given by

$$a_1 = 2\lambda^2(\nu_2^2 + S_0^2 + m_{H_u}^2) + (g_1^2 + g_2^2)\frac{3\nu_1^2 - \nu_2^2}{2}, \tag{20}$$

$$b_1 = 2\lambda^2(2\nu_1\nu_2 - \nu^2 - S_0^2) - (g_1^2 + g_2^2)\nu_1\nu_2, \tag{21}$$

$$c_1 = 4\lambda^2\nu_1 S_0, \qquad c_2 = 4\lambda^2\nu_2 S_0, \tag{22}$$

$$d_1 = 2\lambda^2\mu_0\nu_2, \qquad d_2 = 2\lambda^2\mu_0\nu_1, \tag{23}$$

$$a_2 = 2\lambda^2(\nu_1^2 + S_0^2 + m_{H_d}^2) + (g_1^2 + g_2^2)\frac{3\nu_2^2 - \nu_1^2}{2}, \tag{24}$$

$$e_1 = 2\lambda^2(2\nu_0^2 + \mu_0^2 + m_S^2). \tag{25}$$

Furthermore we have

$$v_1^2 + v_2^2 = \nu_0^2 \quad \text{and} \quad \tan\beta = \frac{v_1}{v_2}, \tag{26}$$

the current experimental EWSB requires $\nu_0 = 247$ GeV.

The eigenvalues of this mass matrix corresponds to the masses of the physical scalars of the model and hence they must be positive. But it is not possible to obtain closed analytical expressions for the eigenvalues.

In the next section we describe how we ensure the positivity constraints of our solution.

4 Phase Transition to a Future Susy Universe

To discuss phase transition from solution (3) to solution (1), we consider the minimization conditions of the scalar potential. For this solution they are

$$S_0(v_1^2 + v_2^2) + \mu_0(v_1v_2 - \nu^2 + \mu_0 S_0) + m_S^2 S_0 = 0, \tag{27}$$

$$v_2(v_1v_2 - \nu^2 + \mu_0 S_0) + v_1 S_0^2 + \frac{v_1}{4\lambda^2}(g_1^2 + g_2^2)(v_1^2 - v_2^2) + m_{H_u}^2 v_1 = 0, \tag{28}$$

$$v_1(v_1v_2 - \nu^2 + \mu_0 S_0) + v_2 S_0^2 + \frac{v_2}{4\lambda^2}(g_1^2 + g_2^2)(v_2^2 - v_1^2) + m_{H_d}^2 v_2 = 0. \tag{29}$$

The Higgs contribution to the vacuum energy for our solution is

$$V(0) = 2\lambda^2[(v_1v_2 - \nu^2 + \mu_0 S)^2 + \nu_0^2 S_0^2 + m_{H_u}^2 v_1^2 + m_{H_d}^2 v_2^2 + m_S^2 S_0^2] + (g_1^2 + g_2^2)\frac{(v_2^2 - v_1^2)^2}{4}. \tag{30}$$

Equations (27)–(29) are a set of coupled nonlinear equations. In the present case where $\tan\beta \neq 1$ is not possible to obtain analytical solutions. Furthermore it is not possible to obtain general expressions for the eigenvalues of the Higgs mass squared matrix.

For a solution to be a true symmetry breaking vacuum:

(i) The Higgs contribution to the vacuum energy must be positive.
(ii) The eigenvalues of the Higgs mass squared which corresponds to the masses of physical scalars of the model must be positive as well.
For an exothermic transition a third condition, namely
(iii) $\nu_0 - \nu > 0$ holds.

But the soft squared masses and the μ_0 and ν are free parameters of the model. Our method was to choose some values for these parameters. Then obtain the values v_1, v_2 and S_0 from (27)–(29). And finally select those solutions which satisfy the positivity constraints (i), (ii), (iii). Our results are presented in Tables 1–3, where we have considered several representative values for the soft Higgs squared masses, μ_0 and ν. We choose both positive and negative values for the soft Higgs masses. We also provide the value of $\tan\beta$ of our solutions. For simplicity in our calculations we have assumed $\lambda = 1$.

5 Conclusions

For simplicity we did not included phases in the Higgs sector. However by the inclusion of soft squared masses we showed that the model has a rich vacuum structure. And we showed that susy-breaking critical points of our extended Higgs model became true minima by addition of these soft terms. This support the current assumption that non-perturbative effects are parameterizable in terms of soft Higgs masses.

For the solution (3) where the values of the scalar fields in the visible sector where equal to their counterparts in the hidden sector we discussed the phase transition and we studied the case of an exothermic transition to an exact susy phase.

Table 1 A set of solutions and soft squared masses. In this table we have assumed $S_0 = 0.025$, $\mu_0 = 2$ and $\nu = 0.99$. Dimensional quantities are given in terms of $\nu_0 = 247$ GeV

v_1	v_2	$m_{H_u}^2$	$m_{H_d}^2$	m_S^2	$\tan\beta$
0.917	0.4	0.157	1.38	44.07	2.29
0.98	0.2	0.03	3.72	57.7	4.90
0.995	0.1	−0.045	8.39	65.44	9.95

Table 2 The second set of solutions and soft squared masses. In this table we have assumed $S_0 = 0.1$, $\mu_0 = 3$ and $\nu = 0.97$

v_1	v_2	$m_{H_u}^2$	$m_{H_d}^2$	m_S^2	$\tan\beta$
0.917	0.4	0.025	0.73	7.50	2.29
0.98	0.2	−0.037	2.33	12.62	4.90
0.995	0.1	−0.082	5.60	15.52	9.95

Table 3 The third set of solutions and soft squared masses. In this table we have assumed $S_0 = 0.05$, $\mu_0 = 4$ and $\nu = 0.98$

v_1	v_2	$m^2_{H_u}$	$m^2_{H_d}$	m^2_S	$\tan\beta$
0.8	0.6	0.17	0.41	21.4	1.33
0.87	0.5	0.12	0.63	25.16	1.74
0.89	0.45	0.1	0.8	27.8	1.98
0.95	0.3	0.04	1.61	36.91	3.17
0.985	0.17	−0.02	3.47	46.15	5.79
0.990	0.14	−0.04	4.46	48.6	7.07
0.996	0.089	−0.07	7.60	52.67	11.2
0.997	0.07	−0.08	9.85	54.16	14.2
0.999	0.045	−0.1	16.11	56.23	22.2

In a more realistic situation it may turn out that the scale of symmetry breaking of the hidden sector be different with that of the visible sector. In this case one expects that the masses of the fields in the visible sector be different from that of the hidden sector [14]. In this situation one should investigate other solutions of this model.

We have provided a partial analysis of soft terms. A more complete assessment of the subject may require the full inclusion of the soft terms [12, 13].

Another direction for further study is to consider the case where all the fields in the scalar sector are complex. We plan to report on these issues in the future.

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